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ANOTHER PROOF OF THE DIFFERENTIABILITY OF A MATRIX (1)

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1. Introduction. Let V be the vector space of all distributions on $(-\infty, \infty)$ and W a translation invariant subspace of finite dimension n. In proving that W is spanned by a set of exponential monomials, ANSELONE and KOREVAAR [1] used the differentiability of a certain matrix and gave four proofs. We give another brief proof. Following the notation of [1], let $\{F_j(t)\}$ span W which is invariant under T_s, $T_s \cdot F_j(t) = F_j(t+s)$. Then let

$$F_j(t+s) = \sum_{k=1}^{n} a_{jk}(s) F_k(t) \quad \text{and} \quad A(s) = (a_{jk}),$$

$A(s)$ is to be shown to be differentiable at 0.

2. Proof of differentiability. Using the imbedding space for distributions constructed in [2], each distribution is an equivalence class of Fundamental Sequences of analytic functions. Let $\{{}_{j,m}f_t(z)\}$ be a representative F. S. S. (Fundamental Sequence) in the equivalence class for $F_j(t)$ and then $\{e^{sz}\,{}_{j,m}f_t(z)\}$ represents $T_s \cdot F_j(t)$. For each m then we have a relation between analytic functions

$$e^{sz}\,{}_{j,m}f_t(z) = \sum_{k=1}^{n} a_{jk}(s)\,{}_{k,m}f_t(z)$$

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and by the linear independence of the analytic (and hence continuous) functions there exist $z_1, \cdots, z_n$ such that

$$ {}_m\hat{f}_t = ({}_{j,m}f_t(z_k)) $$

is non-singular for each m.

If

$$ \mathrm{I}_i = (b_{jk}), \quad b_{jk} = 0, \; j \neq i \\ \qquad = 1, \; j = i $$

we may write

$$ \left[\frac{\mathrm{A}(s) - \mathrm{A}(0)}{s}\right] {}_m\hat{f}_t = \left[\sum_{k=1}^{n} \frac{e^{s \cdot z_k} - 1}{s} \mathrm{I}_k\right] {}_m\hat{f}_t $$

or

$$ \frac{\mathrm{A}(s) - \mathrm{A}(0)}{s} = \sum_{k=1}^{n} \left(\frac{e^{s \cdot z_k} - 1}{s}\right) \mathrm{I}_k . $$

Then taking the limit as $s \to 0$

$$ \mathrm{A}'(0) = \sum_{k=1}^{n} z_k \mathrm{I}_k . $$

Finally since the ${}_{j,m}f_t(z)$ are continuous for z real, it is clear that the z_k could be chosen with zero imaginary parts.

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[2] MYERS, D. E., *An Imbedding Space for Schwartz Distributions,* Pacific J. Math. **11** No. 4 (1961), p. 1467.